# Student Workbook

Unit Review — Exponential & Logarithmic Functions



Name: ______________________ Date: ______________ Class: ________

**ESSENTIAL QUESTION**

How do exponential and logarithmic functions — as inverses of each other — let us model growth, decay, and real-world data, and how do we solve equations involving them?

## Unit Overview

Unit 4 explored exponential and logarithmic functions as a matched pair of inverses. Logarithms undo exponentials: $b^y=x$ is equivalent to $\log_b(x)=y$. The product, quotient, and power rules let us expand or condense logarithmic expressions, which is the key to solving exponential and logarithmic equations. Continuous growth and decay ($A=A0e^{kt}$) model populations, radioactivity, and investments, and linearization/regression let us fit exponential models to real data.

$$b^y = x \iff \log_b(x) = y$$
$$\log_b(MN)=\log_b M + \log_b N$$
$$\log_b(M/N)=\log_b M - \log_b N$$
$$\log_b(M^p)=p\cdot\log_b M$$

| | |
|---|---|
| **Change** | $\log_b(x) = \ln(x)/\ln(b)$ — evaluate any base log with a calculator |
| **Growth** | $A(t) = A0\cdot e^{kt}$; half-life $t(1/2) = \ln(2)/\lvert k\rvert$ |
| **Check** | Every log-equation solution must keep ALL original arguments positive |

## Quick Reference

- To convert between forms: $b^y=x \iff \log_b(x)=y$.
- Exponential graphs have a horizontal asymptote; log graphs have a vertical asymptote — they're reflections of each other over y=x.
- To find k from data: isolate $e^{kt}$, take ln of both sides, then divide by t.

## Worked Examples

**EXAMPLE 1**

**Solve: $3^{x+2} = 81$.**

- Rewrite 81 as a power of 3: $81 = 3^4$
- One-to-one property (same base): $x+2 = 4$

**Answer: x = 2**

**EXAMPLE 2**

**Evaluate $\log_2(32)$ and $\log_5(1/25)$.**

- $\log_2(32)$: $2^5=32$, so $\log_2(32) = 5$
- $\log_5(1/25) = \log_5(5^{-2}) = -2$

**Answer: $\log_2(32) = 5$; $\log_5(1/25) = -2$**

**EXAMPLE 3**

**Expand using log properties: $\log_3((9x^2)/y)$.**

- Quotient rule: $\log_3(9x^2) - \log_3(y)$
- Product rule + power rule: $\log_3(9) + 2\log_3(x) - \log_3(y)$

**Answer: $2 + 2\cdot\log_3(x) - \log_3(y)$**

**EXAMPLE 4**

**Solve: $\log_2(x) + \log_2(x-2) = 3$.**

- Condense (product rule): $\log_2(x(x-2)) = 3$
- Convert to exponential form: $x(x-2) = 8$ -> $x^2-2x-8=0$ -> $(x-4)(x+2)=0$
- Check x=4: $\log_2(4)+\log_2(2)=2+1=3$ ✓. Check x=-2: $\log_2(-2)$ undefined ✗ — extraneous

**Answer: x = 4 (x = -2 is extraneous)**

**EXAMPLE 5**

**A city grows continuously from 20,000 to 26,000 in 5 years. Predict the population after 15 total years.**

- Find k: $26000=20000\cdot e^{5k}$ -> $e^{5k}=1.3$ -> $k=\ln(1.3)/5 \approx 0.05247$
- $A(15) = 20000\cdot e^{0.05247\cdot 15} = 20000\cdot e^{0.7871}$

**Answer: A(15) ≈ 43,930 people**

## Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

**1** Solve: $2^{3x-1} = 32$.

*Hint: Rewrite 32 as a power of 2, then use the one-to-one property.*

**2** Evaluate $\log_4(64)$ and $\log_2(1/8)$.

*Hint: Ask: 4 to what power gives 64? 2 to what power gives 1/8?*

**3** Expand: $\log_5((25x^3)/y^2)$.

*Hint: Apply the quotient rule first, then the product and power rules.*

**4** Solve: $\log_3(x+2) + \log_3(x) = 2$.

*Hint: Condense to one log, convert to exponential form, then use the quadratic formula and check for extraneous roots.*

**5** A radioactive sample decays from 100g to 60g in 8 years. Find k and the half-life.

*Hint: First find k from $60=100\cdot e^{8k}$, then use $t(1/2)=\ln(2)/|k|$.*

## Key Vocabulary

| | |
|---|---|
| **Exponential Function** | A function of the form $f(x)=a\cdot b^x$, with constant ratio between outputs for equal x-steps. |
| **Logarithmic Function** | The inverse of an exponential function: $\log_b(x)=y$ means $b^y=x$. |
| **Product / Quotient Rule** | $\log_b(MN)=\log_b M+\log_b N$; $\log_b(M/N)=\log_b M-\log_b N$. |
| **Power Rule** | $\log_b(M^p) = p\cdot\log_b(M)$ — brings an exponent down as a coefficient. |
| **Change of Base Formula** | $\log_b(x) = \ln(x)/\ln(b)$, used to evaluate any base on a calculator. |
| **Extraneous Solution** | A value that satisfies the transformed equation but not the original — any root making a log argument ≤ 0. |
| **Continuous Growth/Decay** | $A(t)=A0\cdot e^{kt}$; $k>0$ is growth, $k<0$ is decay. |
| **Half-Life** | The time for a decaying quantity to reduce to half its value: $t(1/2)=\ln(2)/|k|$. |

## Check Your Understanding

Circle the letter of the correct answer for each question.

**1** **Solve: $5^{x-1} = 125$.** Multiple Choice

**A** $x = 3$ **B** $x = 4$
**C** $x = 5$ **D** $x = 2$

**2** **Evaluate $\log_3(81)$.** Multiple Choice

- **A** 3
- **B** 4
- **C** 27
- **D** 9

**3** **Expand: $\log_2(8x^3)$.** Multiple Choice

- **A** $3 + 3\cdot\log_2(x)$
- **B** $3\cdot\log_2(8x)$
- **C** $8 + 3\cdot\log_2(x)$
- **D** $24\cdot\log_2(x)$

**4** **Solve: log(x) + log(x-3) = 1.** Multiple Choice

- **A** x = 5
- **B** x = -2
- **C** x = 5 or x = -2
- **D** No solution

**5** **A population grows continuously at rate k=0.04/year. What is the approximate doubling time?** Multiple Choice

- **A** 4 years
- **B** 17.3 years
- **C** 25 years
- **D** 50 years

## Common Mistakes to Avoid

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|---|---|
| ✗ Forgetting to check for extraneous solutions in log equations. | ✓ Always substitute solutions back into the ORIGINAL equation — reject any that make a log argument ≤ 0. |
| ✗ Thinking log(A+B) = log(A) + log(B) — this is NOT a valid property. | ✓ The product rule applies to log(A·B), not log(A+B). There is no rule for the log of a sum. |
| ✗ Forgetting the negative sign on k when modeling decay. | ✓ If the quantity is decreasing, k must be negative — check the sign after solving for k. |
| ✗ Confusing the product rule with the power rule when expanding logs. | ✓ log(MN)=log M+log N (product); $\log(M^p)=p\cdot\log M$ (power) — different operations, different rules. |

## Math Tips

★ To convert between exponential and log form: $b^y=x$ is the same statement as $\log_b(x)=y$.

★ The change of base formula lets you evaluate any log on a calculator: $\log_b(x)=\ln(x)/\ln(b)$.

★ Always verify log-equation solutions in the ORIGINAL equation, not the simplified version.

★ For continuous growth/decay, find k by isolating $e^{kt}$, taking ln of both sides, then dividing by t.

★ Exponential and log graphs are reflections of each other over the line y=x, since they're inverse functions.